

A pit-stop at the
square shop
isn't bad at all

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Selecting the ‘better’ solution to a problem between the first one that comes to mind and the alternative that may follow is not a ‘fait-accompli’. After all, is it better if it is more economical? Or is it better if it is more elegant? Moreover, taking shortcuts does not always lead to a shorter solution. Consider the simple example of a rectangle modified so as to preserve its area.

We put forward two methods for investigating this simple example. The first method that we consider links a rectangle to another with the same area, and to a square of the same area. Our second method of investigation links the two rectangles directly by way of what we refer to as ‘mathematical elegance’. We conclude by outlining the parameters associated with the constructions, as well as comparing the space required for the implementation of these different pathways, in order to make the selection process a little easier.

In Figure 1, the rectangle ABCD is deconstructed to yield the rectangle AB'C'D' of same area and arbitrary side AB'. From rectangle to square the points E, F, G are defined by:

- E: $AE = AB$
 F: $AF = AD$
 G: where AD, or its extension,
 meets the semi-circle of diameter EF.

From square to rectangle they are:

- J: where the perpendicular to B'G
at G meets the extension of BA
D': AD' = AJ

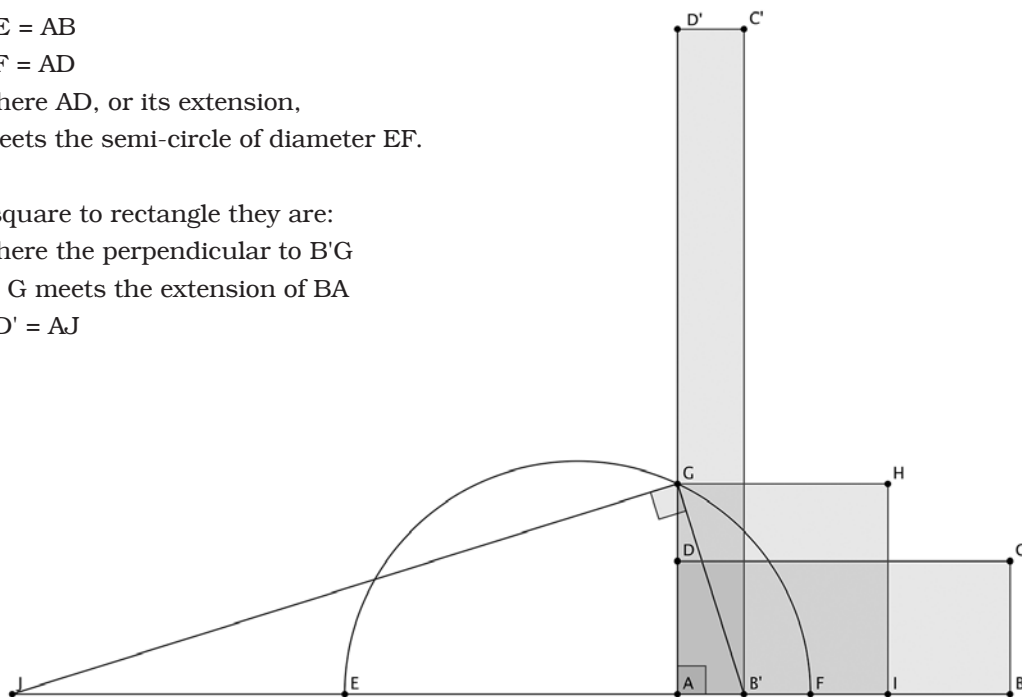


Figure 1. Area of rectangle ABCD = Area of square AIHG = Area of rectangle AB'C'D'.

In Figure 2, the rectangle ABCD is reconstructed from the rectangle AB'C'D' and the point B.

- E': $AE' = AB'$
- F': $AF' = AD'$
- G: where AD', or its extension, meets the semicircle of diameter E'F'
- J': where the perpendicular to BG at G meets the extension of B'A
- D: $AD = AJ'$

In Figure 3, the square is by-passed. It is a little bit like jumping from one rock to the other without landing on the little one in between: it requires a little more momentum and consideration at the start.

Here E, F are as in Figure 1 then:

- K: $AK = AB'$
- D': where AD, or its extension, meets the circle through E, K, F.

The verdict: It is easier to trace a circle through two end-points of one of its diameters than through three arbitrary points, requiring two perpendicular bisectors in the process.

For size, let $AB = a$, $AD = b$, $AB' = a'$, $AD' = b'$, $AG = s$, and denote the radii of the circles in figures 1, 2, and 3 by r , r' , and ρ respectively.

We have :

$$ab = a'b' = s^2$$

$$r = \frac{a+b}{2}, r' = \frac{a'+b'}{2}$$

$$\rho = \sqrt{r^2 + \frac{1}{4}(a'-b')^2} = \sqrt{r'^2 + \frac{1}{4}(a-b)^2}$$

So, $\rho > \max(r, r')$. Hence, requiring more space to construct Figure 3.

A pit-stop at the square shop turns out to be a case of more is less!

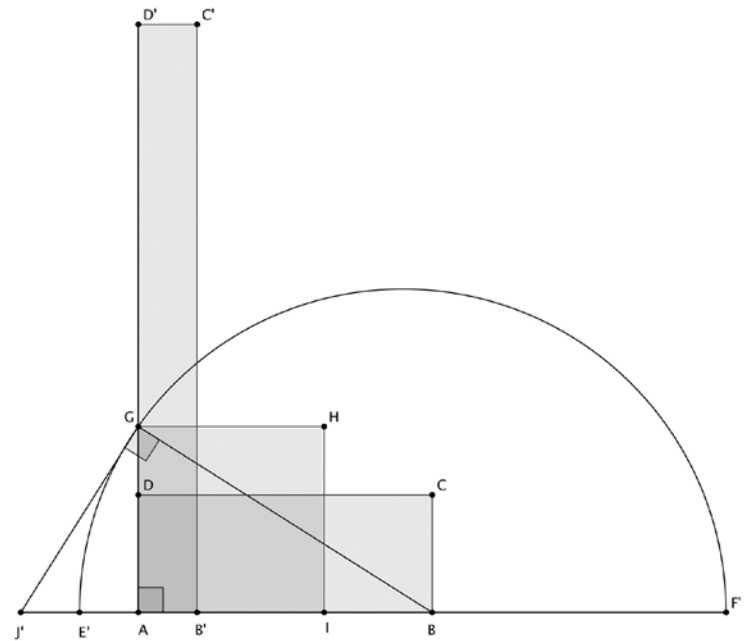


Figure 2. Area of rectangle AB'C'D' = Area of square AIHG = Area of rectangle ABCD.

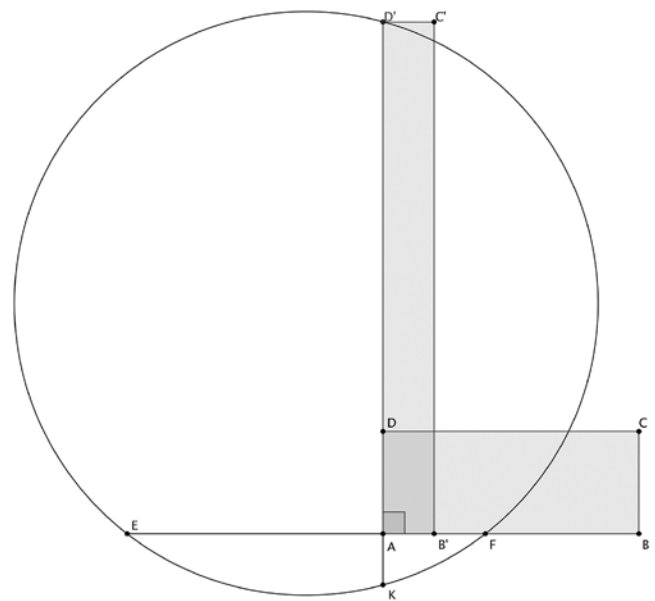


Figure 3. Visually representing the square-free approach.